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Flow of a viscous incompressible fluid between two co-axial unsteadily rotating porous cylinders

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In the present paper an attempt is made to find the solution of the Navier-Stokes equations for the unsteady flow of a viscous incompressible fluid between two co-axial porous cylinders rotating with angular velocities varying periodically with time. A solution has been obtained under the assumption of uniform conditions along the axis of the cylinders. The cylinders being porous, a time-independent hyperbolic cross-flow velocity distribution is superimposed over the circumferential velocity produced due to rotation. It is seen that there is a Bernoulli-type pressure variation in the radial direction. The results are obtained for a particular value of the cross-flow Reynolds number R ($R = -2$). It is observed that in the case of very large frequencies the maxima of circumferential velocity distribution exist in the neighbourhood of the walls. The results transform to the known ones for Couette flow between uniformly rotating co-axial cylinders when $R = 0$ and frequencies are very small.

Introduction

Couette first obtained the exact solution of the Navier-Stokes equations for steady laminar flow of a viscous incompressible fluid between two co-axial rotating cylinders. The solution proved is of importance as it has been used to determine the coefficient of viscosity of a fluid. In a recent paper Sinha & Choudhary (1966) have studied the laminar flow of a viscous incompressible fluid between two co-axial rotating cylinders with uniform radial velocity imposed at the surfaces. They have assumed that the pressure is uniform along the the axis of the cylinders.

In this paper we have discussed the unsteady flow of a viscous liquid between two co-axial porous cylinders rotating with angular velocities varying periodically with time. We have also assumed that pressure is uniform along the axis of the cylinders. The cylinders being porous, a time-independent hyperbolic cross-flow velocity distribution is superimposed over the circumferential velocity produced due to rotation. The results are obtained when the cross-flow Reynolds number R is -2 and we see that they are all finite whereas in the case of Sinha & Choudhary all the results are indeterminate when $R = -2$. The solutions are also obtained in the two extreme cases of small and large frequencies and it is seen that in the case of very large frequencies the maxima of circumferential velocity distribution exist in the neighbourhood of the walls. When $R = 0$ and frequencies are very small, the results transform to the known ones for Couette fllow between two uniformly rotating co-axial cylinders.

NOTATIONS

ρ = density of the fluid
x = axial coordinate
r = radial coordinate
ϕ = azimuthal coordinate
u = axial velocity
v = cross-flow velocity
w = azimuthal velocity
P = pressure
μ = coefficient of viscosity
ν = kinematic viscosity
r_1= radius of the inner cylinder
r_2= radius of the outer cylinder
v_1= cross-flow velocity at the wall of the inner cylinder
v_2= cross-flow velocity at the wall of the outer cylinder

1. *Equations of Motion and their Solution*

The Navier-Stokes equations of motion of a viscous incompressible fluid in cylindrical polar coordinates are (Pai 1956)

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{w}{r}\frac{\partial v}{\partial \phi} - \frac{w^2}{r} + u\frac{\partial v}{\partial x}$$
$$= -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \phi^2} - \frac{2}{r^2}\frac{\partial w}{\partial \phi} + \frac{\partial^2 v}{\partial x^2}\right] \quad \ldots(1.1)$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial r} + \frac{w}{r}\frac{\partial w}{\partial \phi} + \frac{vw}{r} + u\frac{\partial w}{\partial x}$$
$$= -\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \phi} + \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{w}{r^2} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v}{\partial \phi} + \frac{\partial^2 w}{\partial x^2}\right], \quad \ldots(1.2)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{w}{r}\frac{\partial u}{\partial \phi} + u\frac{\partial u}{\partial x}$$
$$= -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial x^2}\right], \quad \ldots(1.3)$$

and the equation of continuity is

$$\frac{\partial v}{\partial r} + \frac{v}{r} + \frac{1}{r}\frac{\partial w}{\partial \phi} + \frac{\partial u}{\partial x} = 0. \qquad \ldots(1.4)$$

For flow between two rotating porous cylinders

$$\frac{\partial}{\partial \phi}\Big(\quad\Big) = 0 \text{, for axial symmetry;}$$

$$u = 0, \text{ for motion due to rotation only.}$$

Under these conditions, equations (1.1), (1.2), (1.3) and (1.4) reduce to

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial x^2}\right], \qquad \ldots(1.5)$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial r} + \frac{vw}{r} = \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{w}{r^2} + \frac{\partial^2 w}{\partial x^2}\right], \qquad \ldots(1.6)$$

$$\frac{\partial P}{\partial x} = 0, \qquad \ldots(1.7)$$

and

$$\frac{\partial v}{\partial r} + \frac{v}{r} = 0. \qquad \ldots(1.8)$$

Equation (1.7) states the condition of uniform pressure distribution along the axis of the cylinders.

Assuming $\frac{\partial v}{\partial x} = 0$, for uniforn suction and injection throughout the whole length and $\frac{\partial w}{\partial x} = 0$, for circumferential velocity produced due to rotation only, equations (1.5), (1.6) and (1.8) reduce to

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r}\right] \qquad \ldots(1.9)$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial r} + \frac{vw}{r} = \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{w}{r^2}\right], \qquad \ldots(1.10)$$

and

$$\frac{\partial v}{\partial r} + \frac{v}{r} = 0. \qquad \ldots(1.11)$$

We now assume that the suction rate at one wall is equal to the injection rate at the other wall. Therefore

$$r_2 v_2 = r_1 v_1, \qquad \ldots(1.12)$$

where v_1, v_2 are the cross-flow velocities at the walls of the smaller and larger tubes, respectively. Then from equations (1.11) and (1.12), we get

$$vr = v_1 r_1 = v_2 r_2. \qquad \ldots(1.13)$$

From equations (1.9) and (1.11), assuming cross-flow velocity to be independent of time, we get

$$\rho\left(\frac{v^2 + w^2}{r}\right) = \frac{\partial P}{\partial r}. \qquad \ldots(1.14)$$

Equation (1.14) states Bernoulli-type pressure variation in the radial direction. Now equation (1.10) becomes

$$\frac{\partial^2 w}{\partial r^2} + \frac{(1-R)}{r}\frac{\partial w}{\partial r} - \frac{(1+R)}{r^2}\,.\,w = \frac{1}{\nu}\frac{\partial w}{\partial t}, \qquad \ldots(1.15)$$

where $R = \frac{r_1 v_1}{\nu}$ is cross-flow Reynolds number.

We now put $w = f(r)e^{int}$. (1.16)

into (1.15) and take the real part of the final solution as the required result. Substituting equation (1.16) into equation (1.15), we have

$$\frac{d^2 f}{dr} + \frac{(1-R)}{r}\frac{df}{dr} - \left[\frac{in}{\nu} + \frac{(1+R)}{r^2}\right] f = 0. \qquad (1.17)$$

The boundary conditions for w are

$w = \Omega_1 r_1 e^{int}$ when $r = r_1$, and $w = \Omega_2 r_2 e^{int}$ when $r = r_2$, where only the real parts are taken.

Therefore the boundary conditions for f are

$$\left.\begin{aligned} &f = \Omega_1 r_1 \text{ when } r = r_1, \\ \text{and} \quad &f = \Omega_2 r_2 \text{ when } r = r_2. \end{aligned}\right\} \qquad \ldots(1.18)$$

The general solution of equation (1.17) under the boundary conditions (1.18) is

$$f = r^{R/2}\left[\Omega_1 r_1^{(2-R)/2} \frac{\{J_m(rpi^{3/2})\,Y_m(r_2 pi^{3/2}) - J_m(r_2 pi^{3/2})\,Y_m(rpi^{3/2})\}}{J_m(r_1 pi^{3/2})\,Y_m(r_2 pi^{3/2}) - J_m(r_2 pi^{3/2})\,Y_m(r_1 pi^{3/2})} + \Omega_2 r_2^{(2-R)/2} \frac{\{J_m(r_1 pi^{3/2})\,Y_m(rpi^{3/2}) - J_m(rpi^{3/2})\,Y_m(r_1 pi^{3/2})\}}{J_m(r_1 pi^{3/2})\,Y_m(r_2 pi^{3/2}) - J_m(r_2 pi^{3/2})\,Y_m(r_1 pi^{3/2})}\right]$$

Or

$$f = r^{R/2}\left[\Omega_1 r_1^{(2-R)/2}\,\frac{\{J_m(rpi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(rpi^{3/2})\}}{J_m(r_1pi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(r_1pi^{3/2})}\right.$$

$$\left.+\,\Omega_2 r_2^{(2-R)/2}\,\frac{\{J_m(r_1pi^{3/2})\,J_{-m}(rpi^{3/2}) - J_m(rpi^{3/2})\,J_{-m}(r_1pi^{3/2})\}}{J_m(r_1pi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(r_1pi^{3/2})}\right]$$

according as m is an integer or not, where $m = \pm\frac{R+2}{2}$ and $p = (n/\nu)^{1/2}$.

Therefore the circumferential velocity is

$$w = R_e\,(r)^{R/2}\left[\Omega_1 r_1^{(2-R)/2}\,\frac{\{J_m(rpi^{3/2})\,Y_m(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,Y_m(rpi^{3/2})\}}{J_m(r_1pi^{3/2})\,Y_m(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,Y_m(r_1pi^{3/2})}\right.$$

$$\left.+\,\Omega_2 r_2^{(2-R)/2}\,\frac{\{J_m(r_1pi^{3/2})\,Y_m(rpi^{3/2}) - J_m(rpi^{3/2})\,Y_m(r_1pi^{3/2})\}}{J_m(r_1pi^{3/2})\,Y_m(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,Y_m(r_1pi^{3/2})}\right]e^{int},$$

Or

$$w = R_e\,(r)^{R/2}\left[\Omega_1 r_1^{(2-R)/2}\,\frac{\{J_m(rpi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(rpi^{3/2})\}}{J_m(r_1pi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(r_1pi^{3/2})}\right.$$

$$\left.+\,\Omega_2 r_2^{(2-R)/2}\,\frac{\{J_m(r_1pi^{3/2})\,J_{-m}(rpi^{3/2}) - J_m(rpi^{3/2})\,J_{-m}(r_1pi^{3/2})\}}{J_m(r_1pi^{3/2})\,J_{-m}(r_2pi^{3/2}) - J_m(r_2pi^{3/2})\,J_{-m}(r_1pi^{3/2})}\right]e^{int}$$

according as m is an integer or not. In the above expressions R_e denotes the real part.

Now the circumferential velocity when $R = -2$ is

$$wr = R_e\left[\Omega_1 r_1^2\,\frac{\{J_0(rpi^{3/2})\,Y_0(r_2pi^{3/2}) - J_0(r_2pi^{3/2})\,Y_0(rpi^{3/2})\}}{J_0(r_1pi^{3/2})\,Y_0(r_2pi^{3/2}) - J_0(r_2pi^{3/2})\,Y_0(r_1pi^{3/2})}\right.$$

$$\left.+\,\Omega_2 r_2^2\,\frac{\{J_0(r_1pi^{3/2})\,Y_0(rpi^{3/2}) - J_0(rpi^{3/2})\,Y_0(r_2pi^{3/2})\}}{J_0(r_1pi^{3/2})\,Y_0(r_2pi^{3/2}) - J_0(r_2pi^{3/2})\,Y_0(r_1pi^{3/2})}\right]e^{int} \qquad \ldots(1.19)$$

The expression (1.19) is finite whereas the expression for the circumferential velocity obtained by Sinha & Choudhury takes indeterminate form for $R = -2$

2 *Velocity Distribution for Small Frequencies*

For small frequencies, that is n is small, we have

$$J_0(rpi^{3/2}) \simeq 1 + \frac{ir^2p^2}{4}$$

and

$$Y_0(rpi^{3/2}) \simeq \frac{2}{\pi}\left[\left(\gamma + \log\frac{rpi^{3/2}}{2}\right)\left(1 + \frac{ir^2p^2}{2}\right) - \frac{ir^2p^2}{4}\right],$$

where γ is Euler's constant.

Hence

$$J_0(r_1 p i^{3/2})\, Y_0(r_2 p i^{3/2}) - J_0(r_2 p i^{3/2})\, Y_0(r_1 p i^{3/2})$$

$$= \frac{2}{\pi}\left[\log\left(\frac{r_2}{r_1}\right) + \frac{ip^2}{4}\left\{(r_1^2+r_2^2)\log\left(\frac{r_2}{r_1}\right) - (r_2^2 - r_1^2)\right\}\right].$$

Substituting these values in (1.19), we get

$$wr = \frac{\Omega_1 r_1^2 \log\left(\frac{r_2}{r}\right) + \Omega_2 r_2^2 \log\left(\frac{r}{r_1}\right)}{\log\left(\frac{r_2}{r_1}\right)} \cos nt. \qquad \text{.. (2.1)}$$

3. *Velocity Distribution for Large Frequencies*

When n is large, taking asymptotic expansions of Bessel functions, $rp>10$ aud $-\frac{\pi}{2} \leqslant$ phase $(rpi^{3/2}) \leqslant \frac{\pi}{2}$, we have

$$J_0(rpi^{3/2}) \simeq \left(\frac{2}{\pi r p i^{3/2}}\right)^{1/2} \cdot \cos\left(rpi^{3/2} - \frac{\pi}{4}\right)$$

and

$$Y_0(rpi^{3/2}) \simeq \left(\frac{2}{\pi r p i^{3/2}}\right)^{1/2} \cdot \sin\left(rpi^{3/2} - \frac{\pi}{4}\right).$$

Substituting these values in (1.19), we have

$$wr = \Omega_1 r_1^2 \left(\frac{r_1}{r}\right)^{1/2} . e^{-(r-r_1)p/\sqrt{2}} \cos\left\{(r - r_1)p/\sqrt{2} - nt\right\}$$

$$+ \Omega_2 r_2^2 \left(\frac{r_2}{r}\right)^{1/2} . e^{-(r_2 - r)p/\sqrt{2}} \cos\left\{(r_2 - r)p/\sqrt{2} - nt\right\} \qquad \text{...(3.1)}$$

Hence when n is large, it is found that maxima of circumferential velocity distribution exist in the neighbourhood of the walls.

4. *Results When the Inner Cylinder is at Rest*

The case, when the inner cylinder is at rest and the outer rotates, has some practical importance. The circumferential velocity when the inner cylinder is at rest and $R = -2$

$$wr = R_e\left[\frac{\Omega_2 r_2^2\{J_0(r_1 p i^{3/2})\, Y_0(r p i^{3/2}) - J_0(r p i^{3/2})\, Y_0(r_1 p i^{3/2})\}}{J_0(r_1 p i^{3/2})\, Y_0(r_2 p i^{3/2}) - J_0(r_2 p i^{3/2})\, Y_0(r_1 p i^{3/2})}\right] e^{int} \qquad \text{...(4.1)}$$

Now from (4.1) the expressions for the circumferential veloclty for small and large frequencies can easily be obtained.

Using the recurrence relation

$$- J_0(z)\, Y_1(z) + J_1(z)\, Y_0(z) = \frac{2}{\pi z}$$

we get the shearing stress at the inner cylinder from (4.1) as

$$\tau = \mu\left(\frac{\partial w}{\partial r}\right)_{r=r_1} = R_e\left[\frac{2\mu}{\pi}\cdot\frac{\Omega_2 r_2{}^2}{r_1{}^2}\cdot\right.$$

$$\left.\frac{1}{J_0\,(r_1 p i^{3/2})\, Y_0\,(r_2 p i^{3/2}) - J_0\,(r_2 p i^{3/2})\, Y_0(r_1 p i^{3/2})}\right] e^{int}. \quad \text{.. (4.2)}$$

The shearing stress for small frequencies is

$$\tau = -\frac{\mu\Omega_2 r_2{}^2}{r_1{}^2 \log\left(\dfrac{r_2}{r_1}\right)} \cos nt. \quad \text{...(4.3)}$$

For large frequencies the shearing stress is

$$\tau = p\mu\Omega_2\cdot\left(\frac{2r_2{}^5}{r_1{}^3}\right)^{1/2} \cdot e^{-(r_2-r_1)p/\sqrt{2}}$$

$$[\cos\{(r_2 - r_1)p/\sqrt{2} - nt\} + \sin\{(r_2 - r_1)p\sqrt{2} - nt\}] \quad \text{...(4.4)}$$

From (4.4) it is clear that the shearing stress at the inner cylinder will be very small in the case of very large frequencies.

The torque transmitted by the fluid to unit length of the inner cylinder is

$$M = R_e\left[\frac{4\mu\Omega_2 r_2{}^2}{J_0\,(r_1 p i^{3/2})\, Y_0(r_2 p i^{3/2}) - J_0\,(r_2 p i^{3/2})\, Y_0\,(r_1 p i^{3/2})}\right] e^{int}. \quad \text{...(4.5)}$$

From (4.5) the expressions for the torque transmitted by the fluid to unit length of the inner cylinder for small and large frequencies can be easily derived.

5. *Solutions for the case $R = 0$*

The circumferential velocity when $R = 0$ is

$$w_0 = R_e\left[\Omega_1 r_1\,\frac{\{J_1(rpi^{3/2})\, Y_1(r_2 p i^{3/2}) - J_1(r_2 p i^{3/2})\, Y_1(rpi^{3/2})\}}{J_1(r_1 p i^{3/2})\, Y_1(r_2 p i^{3/2}) - J_1(r_2 p i^{3/2})\, Y_1(r_1 p i^{3/2})}\right.$$

$$\left. + \Omega_2 r_2\,\frac{\{J_1(r_1 p i^{3/2})\, Y_1(rpi^{3/2}) - J_1(rpi^{3/2})\, Y_1(r_1 p i^{3/2})\}}{J_1(r_1 p i^{3/2})\, Y_1(r_2 p i^{3/2}) - J_1(r_2 p i^{3/2})\, Y_1(r_1 p i^{3/2})}\right] e^{int}. \quad \text{...(5.1)}$$

For small frequencies

$$J_1\,(r_1 p i^{3/2})\, Y_1(r_2 p i^{3/2}) - J_1(r_2 p i^{3/2})\, Y_1(r_1 p i^{3/2}) = \frac{(r_2{}^2 - r_1{}^2)}{\pi r_1 r_2},$$

Therefore the circumferential velocity for small frequencies is

$$w_0 = \frac{1}{(r_2^2 - r_1^2)}\left[r\,(\Omega_2 r_2^2 - \Omega_1 r_1^2) - \frac{r_1^2\, r_2^2}{r}(\Omega_2 - \Omega_1)\right]\cos nt. \qquad \dots(5.2)$$

The expression (5.2) is as in the Couette flow between two uniformly rotating cylinders with solid surfaces ($R = 0$).

For large frequencies we have

$$J_1\,(rpi^{3/2}) \simeq \left(\frac{2}{\pi r p i^{3/2}}\right)^{1/2} \cdot \cos\left(rpi^{3/2} - \frac{3\pi}{4}\right)$$

and

$$Y_1\,(rpi^{3/2}) \simeq \left(\frac{2}{\pi r p i^{3/2}}\right)^{1/2} \cdot \sin\left(rpi^{3/2} - \frac{3\pi}{4}\right).$$

Substituting these values in (5 1), we obtain

$$w_0 = \Omega_1\left(\frac{r_1^3}{r}\right)^{1/2} \cdot e^{-(r-r_1)p/\sqrt{2}} \cdot \cos\left\{(r-r_1)\,p/\sqrt{2} - nt\right\}$$
$$+\,\Omega_2\left(\frac{r_2^3}{r}\right)^{1/2} e^{-(r_2-r)p/\sqrt{2}}. \cos\left\{(r_2-r)\,p/\sqrt{2} - nt\right\}. \qquad (5.3)$$

Hence when n is large, it is again found that maxima of circumferential velocity distribution exist in the neighbourhood of the walls.

Now the circumferential velocity when the inner cylinder is at rest and $R = 0$ is

$$w_1 = R_e\left[\Omega_2 r_2 \frac{\{J_1(r_1pi^{3/2})\,Y_1(rpi^{3/2}) - J_1(rpi^{3/2})\,Y_1(r_1pi^{3/2})\}}{J_1(r_1pi^{3/2})\,Y_1(r_2pi^{3/2}) - J_1(r_2pi^{3/2})\,Y_1(r_1pi^{3/2})}\right]e^{int}. \qquad \dots(5.4)$$

Using the recurrence relation

$$-J_1(z)\,Y_2(z) + J_2(z)\,Y_1(z) = \frac{2}{\pi z}$$

we get the shearing stress at the inner cylinder from (5.4) as

$$\tau_0 = R_e\left[\frac{2\mu}{\pi} \cdot \frac{\Omega_2 r_2}{r_1} \frac{1}{J_1(r_1pi^{3/2})\,Y_1(r_2pi^{3/2}) - J_1(r_2pi^{3/2})\,Y_1(r_1pi^{3/2})}\right]e^{int}. \qquad \dots(5.5)$$

The shearing stress for small frequencies is

$$\tau_0 = \frac{2\mu\Omega_2 r_2^2}{(r_2^2 - r_1^2)}\cos nt. \qquad \dots(5.6)$$

The expression (5.6) for shearing stress is as in the Couette flow between two uniformly rotating cylinders with solid surfaces.

For large frequencies the shearing stress is

$$\tau_\eta = \mu p \Omega_2 \left(\frac{2r_2^3}{r_1} \right)^{1/2} . e^{-(r_2 - r_1) p/\sqrt{2}} . \left[\cos \left\{ (r_2 - r_1) p/\sqrt{2} - nt \right\} \right.$$

$$\left. + \sin \left\{ (r_2 - r_1) p/\sqrt{2} - nt \right\} \right] \quad ...(5.7)$$

From (5.7) it is obvious that the shearing stress at the inner cyliner will be very small in the case of very large frequencies.

Now the torque transmitted by the fluid to unit length of the inner cylinder is

$$M_0 = R_e \left[\frac{4\mu\Omega_2 r_1 r_2}{J_1(r_1 p i^{3/2})\, Y_1(r_2 p i^{3/2}) - J_1(r_2 p i^{3/2})\, Y_1(r_1 p i^{3/2})} \right] e^{int}. \quad .. (5.8)$$

For small frequencies the above expression gives

$$M_0 = \frac{4\mu\pi\Omega_2 r_1^2 r_2^2}{(r_2^2 - r_1^2)} \cos nt.$$

The expression (5.8) for large frequencies is

$$M_0 = 2\sqrt{2\mu\pi p}\,\Omega_2 r_1^{3/2} r_2^{3/2} .\ e^{-(r_2 - r_1)p/\sqrt{2}}.$$

$$\left[\cos \left\{ (r_2 - r_1) p/\sqrt{2} - nt \right\} + \sin \left\{ (r_2 - r_1) p/\sqrt{2} - nt \right\} \right],$$

which will also be very small for very large frequencies

Concusions

Sinha & Choudhary have made the calculations for small values of $R\,(-1 \leqslant R \leqslant 1)$ and results obtained by them become indeterminate when $R = -2$, but in the present paper we have not imposed any restriction upon R and it is also seen that the results can be obtained for all possible values of R including $R = -2$.

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